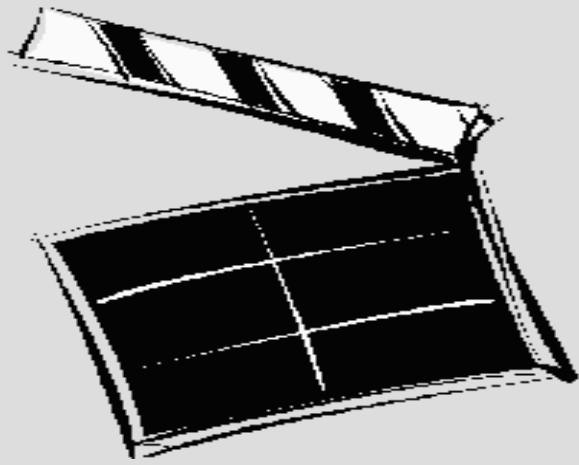


Inequality in Life Satisfaction and the EU Debt Crisis.

**Review of Measurement Approaches for
Ordinal Data and Empirical Assessment of
their Suitability.**

Agenda

1. Motivation
2. Assessing Inequality (with ordinal data)
 - a) Dominance Criteria
 - b) Measures
3. Application – Subjective Wellbeing in Europe
4. Conclusion



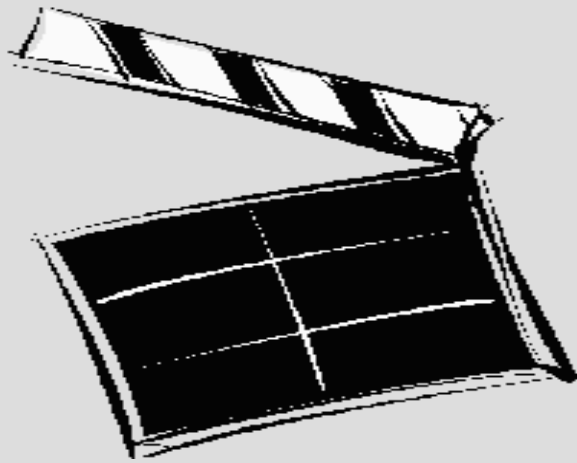
Motivation

Motivation

- Increasing Income Inequality since 1970s (Piketty, 2014)
- Income-Happiness-Relationship (Easterlin-Paradox)
- However: Decreasing Inequality in Happiness since the 1970s
 - Veenhoven (2005), Stevenson/Wolfers (2008), Clark et al. (2014), Dutta/Foster (2013), Madden (2011)
 - Implications for Cohesion goals in the EU-treaty?
- Questionable Methods (Cardinal vs. Ordinal)
 - notable exceptions Dutta/Foster (2013), Madden (2011)

Motivation

- Study's contributions:
 - Methodological
 - Extensive Cross-disciplinary Review of approaches
 - Comparing and Categorizing measures by properties
 - Empirical
 - effects of the EU-crisis on subjective wellbeing?
 - Bootstrapped confidence intervals
 - „feedback“: Applicability of methods?



Assessing Inequality with ordinal data

The Problem

- Blair/Lacy (2000)
 - Why specific „ordinal dispersion measures“?
 - Ubiquity of ordinal data in (social) science
 - Problems of conventional/standard measures
(such as Index of Qualitative Variation, or Variance)

scale	1	2	3	4	5	Variance
n (Male)	20	20	20	20	60	2.06
n (Female)	30	10	25	45	30	2.26

scale	In1	In2	In3	In4	In5	Variance
n (Male)	20	20	20	20	60	0.35
n (Female)	30	10	25	45	30	0.32

Ways to deal with the problem

Rankings/Orderings

Robustness

Completeness

Dominance
Criteria

Ordinal
Measures

Cardinalization
Approaches

e.g.
Allison/Foster (2004),
Madden (2011)

e.g.
Berry/Mielke (1992)
Blair/Lacy (2000)
Van der Eijk (2001)
Abul Naga/Yalcin (2008)

e.g.
Kalmijn/Veenhoven (2005)
Van Doorslaer et al. (2002)

First order dominance

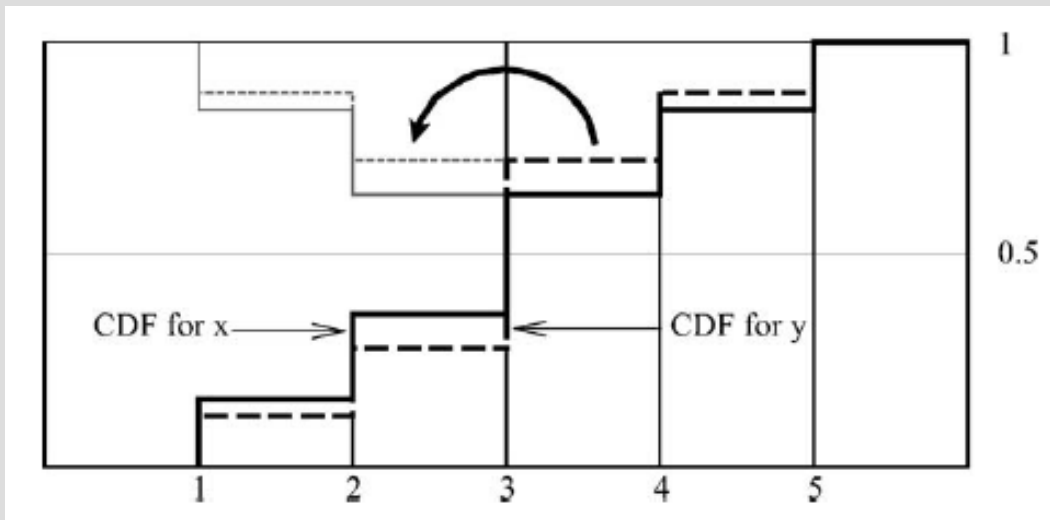
- Analogous to the cardinal case
 - Distribution $x = (x_1, x_2, x_3, \dots, x_n)$ first order dominates distribution $y = (y_1, y_2, y_3, \dots, y_n)$ if $X_k \leq Y_k$ for all categories $k = 1, \dots, n$

(Note: $X_k = \frac{F_x(k)}{X}$ are population shares)

- first order dominance implies mean-based orderings to be unambiguous (scale-independent)
 - Levels vs. Distribution!

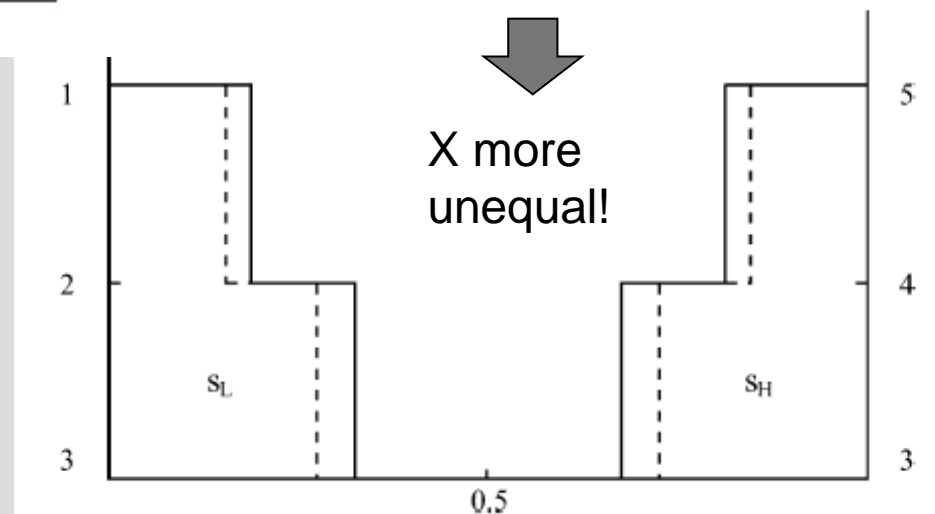
Second order dominance?

The Allison-Foster (2004) „breakthrough“



„median preserving spread“

But: only partial ordering



Cardinalization Approaches

- Simple Cardinalization
 - Kalmijn and Veenhoven (2005) interpreting Van Praag (1991): direct one to one conversion of numerical identifiers
 - Misinterpretation Van Praag?!
similarly critical: Studer (2011) and (!) Veenhoven (2009)
 - Sophisticated Cardinalization
 - Van Doorslaer et al. (2002) compare various methods: interval regression superior alternative
 - Critique: Van Doorslaer/Jones (2003), Costa-Font/Hernández-Quevedo (2013)
- Apply Standard Measures to these data
- standard deviation: Kalmijn and Veenhoven (2005:33) “advise researchers to continue the use of this statistic“
 - Gini etc.

Ordinal Measures

- “No suitable statistic ordinal index [..] has been proposed yet [..]”(Kalmijn and Arends, 2010, p. 155)
- Based on notion of median-preserving spread (Allison/Foster, 2004): Apouey (2007), Abul Naga and Yalcin 2008, 2010)
- Ranks-based approaches: Giudici/Raffinetti (2011), Cowell/Flachaire (2012)

Ordinal Measures

Kobus (2014) transfers classical Atkinson-result into ordinal framework:

All measures satisfying the median-preserving spread principle will yield the same ranking if AF-ordering applies

→ Family of measures

Absolute Value and Median-based Measures

- Berry/Mielke (1992), Blair/Lacy (2000)
- Abul Naga/Yalcin (2008, 2010), Apouey (2007)
- Van der Eijk (2001)
- ~~Tastle/Wierman (2007)~~

Ordinal Measures (cont'd)

Other measures that have been developed just recently:

Rank-based Measures

- Cowell/Flachaire (2012)
- Giudici/Raffinetti (2011)

- Blair/Lacy (2000)
 - Develop a group of ordinal measures :
 - Cumulative Frequencies contains all Information
→ensures scale-invariance
 - Equality not uniquely defined in ordinal case
 - Maximum Dispersion (=Polarization) is unique
 - Dispersion: $d^2 = \sum_{i=1}^{k-1} (X_i - \frac{1}{2})^2$
→Percentage of maximum ploarization dispersion:
$$I_{BL} = 1 - \frac{d^2}{d_{max}^2}$$

(equivalent to IOV defined by Berry/Mielke, 1992)
 - Dispersion vs. Inequality (vs. Polarization)!?
→Allison-Foster-Ordering

- Abul Naga/Yalcin (2008) and Apouey (2007) independently derive measures axiomatically, both in a
 - General Form and
 - Absolute Value Measures (AVM) as a subset
- But only AVM are median-independent (Abul Naga/Yalcin, 2010)!

- $I_{BL} = 1 - \frac{d^2}{d_{max}^2}$ one such AVM

- Similarly to the approach of equivalence classes in Abul Naga/Yalcin (2010), Van der Eijk (2010) defines measure of Agreement:

$$A = U \cdot \left(1 - \frac{(S-1)}{(K-1)} \right) \text{ with } U = \frac{(K-2) \cdot TU - (K-1) \cdot TDU}{(K-2) \cdot (TU + TDU)}$$

- Cowell/Flachaire (2012) criticize median as point of reference (not generally well-defined in categorical data)

Develop axiomatic approach based on the notion of „status“
(→ utility implied by status as an alternative to measuring utility directly especially useful for ordinal data)

Two main steps:

1. Define individual Status

$$s_i = \psi(u_i, F(\cdot), n)$$

„Merger Principle“ implies additive structure

→ upward- vs. downward-looking status

2. Aggregate

$$I_\alpha(\mathbf{s}; e) = \frac{1}{\alpha[\alpha - 1]} \left[\frac{1}{n} \sum_{i=1}^n \left[\frac{s_i}{e} \right]^\alpha - c(e) \right]$$

Rank-Based I (cont'd)

- With:
 - α : sensitivity parameter (different parts of the distribution)
 - e : reference point (mean?, median?)
 - c : constant conditional on e
- maximum as superior reference point ($e=1$)
- For individual relative status to be an affine transformation of utility, application of Eichhorn's (1978) Theorem yields:

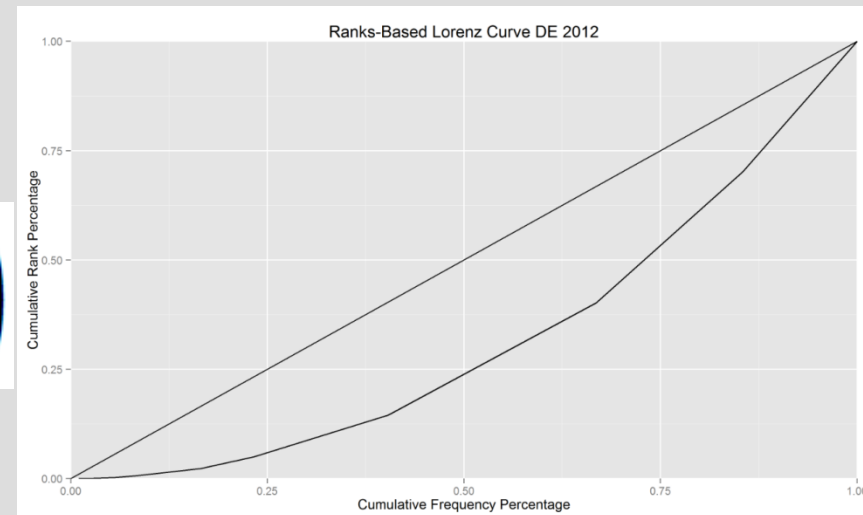
$$I_{\alpha}(\mathbf{s}, 1) = \begin{cases} \frac{1}{\alpha(\alpha-1)} \left[\frac{1}{n} \sum_{i=1}^n s_i^{\alpha} - 1 \right], & \text{if } \alpha \neq 0, 1, \\ -\frac{1}{n} \sum_{i=1}^n \log s_i. & \text{if } \alpha = 0. \end{cases}$$

- choice of α ?

- Giudici/Raffinetti (2011) suggest a Lorenz-Curve based on Ranks from the ordinal distribution

- Coordinates:

$$(X(\cdot), Q(r)) = \left(\frac{\sum_{j=1}^i n_j}{\sum_{j=1}^k n_j}, \frac{\sum_{j=1}^i r_j n_j}{\sum_{j=1}^k r_j n_j} \right)$$



- Trapezoid-Rule yields Ranks-Based Gini:

$$I_{GR} = 1 - \sum_{r=1}^k (X(r-1) + X(r))(F(r) - F(r-1))$$

Digression: Ontological Considerations

- Kobus (2014): Pigou-Dalton-Transfer meaningless
 - But mixes up ontological and methodological Arguments
 - Methodological: impact of the PD-transfer may remain indeterminate (Income on ordinal scale?)
 - Ontological: transferring health/happiness from a healthy to a less healthy individual mean

- Kalmijn/Veenhoven (2005):

- Capacity Variables



Summation feasible (e.g. volume)



Quantity Interpretation (and frequentist interpretation)

vs.

Intensity Variables

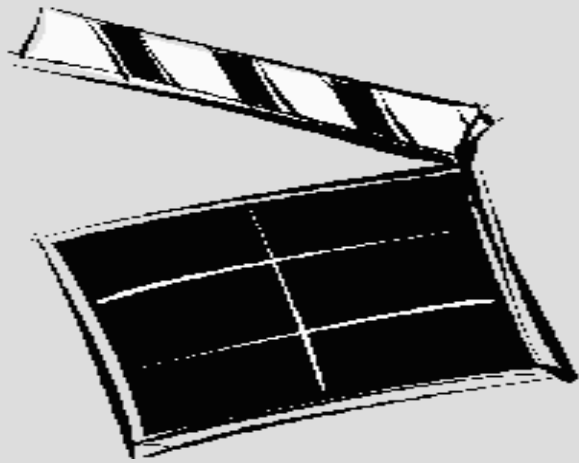


Summation not feasible (e.g. temperature)



Only frequentist interpretation

- Boundedness of Scale



Application – Europe 2002-2012

Data – European Social Survey (ESS)

- Survey-Data, biannually since 2002 (i.e. 2012 = 6th Wave)
- Administration: Centre for Comparative Social Surveys at City University London (Rory Fitzgerald), financial support by 5th/6th Framework-Program of the Europ. Commission, Europ. Research Fund and national Funds
- Idea: Establish Data Basis for Research on “attitudes, beliefs and behaviour patterns of diverse populations”
- 36 Participating Countries, BUT not a balanced “Panel” (e.g. AT only Waves 1-3 and 6)

Data – European Social Survey (ESS)

- Sampling shall ensure Representativity for Inhabitants over 15 years of age, irrespectively of origin
- Sampling (and partly specification of questionnaire (e.g. Income Categories) administred by Partners in Participating Countries (generally National Statistics Offices)
- Guideline with respect to Sample Size: 1500 Obs. Per Country and Wave (for Countries <2 Mio. Inhabitants at least 800 Obs.)

Dominance Analysis

- Dominance Comparisons

	Pairs of Years (for each country)	Pairs of Countries (for each period)
Total no. of pairs	270	432
First order dominance	39 (14%)	151 (35%)
Allison-Foster dominance	1	2

- First order dominance for DK vs. NL only in 2002-2008
 - But: Mean always greater in DK → Ordering by means (and hence mean-based inequality measures) is misleading not only in theory
- AF-dominance:
 - NL: 2008 less unequal than 2002
 - 2010: Inequality in ES > Inequality in DE, GB

Dominance Analysis

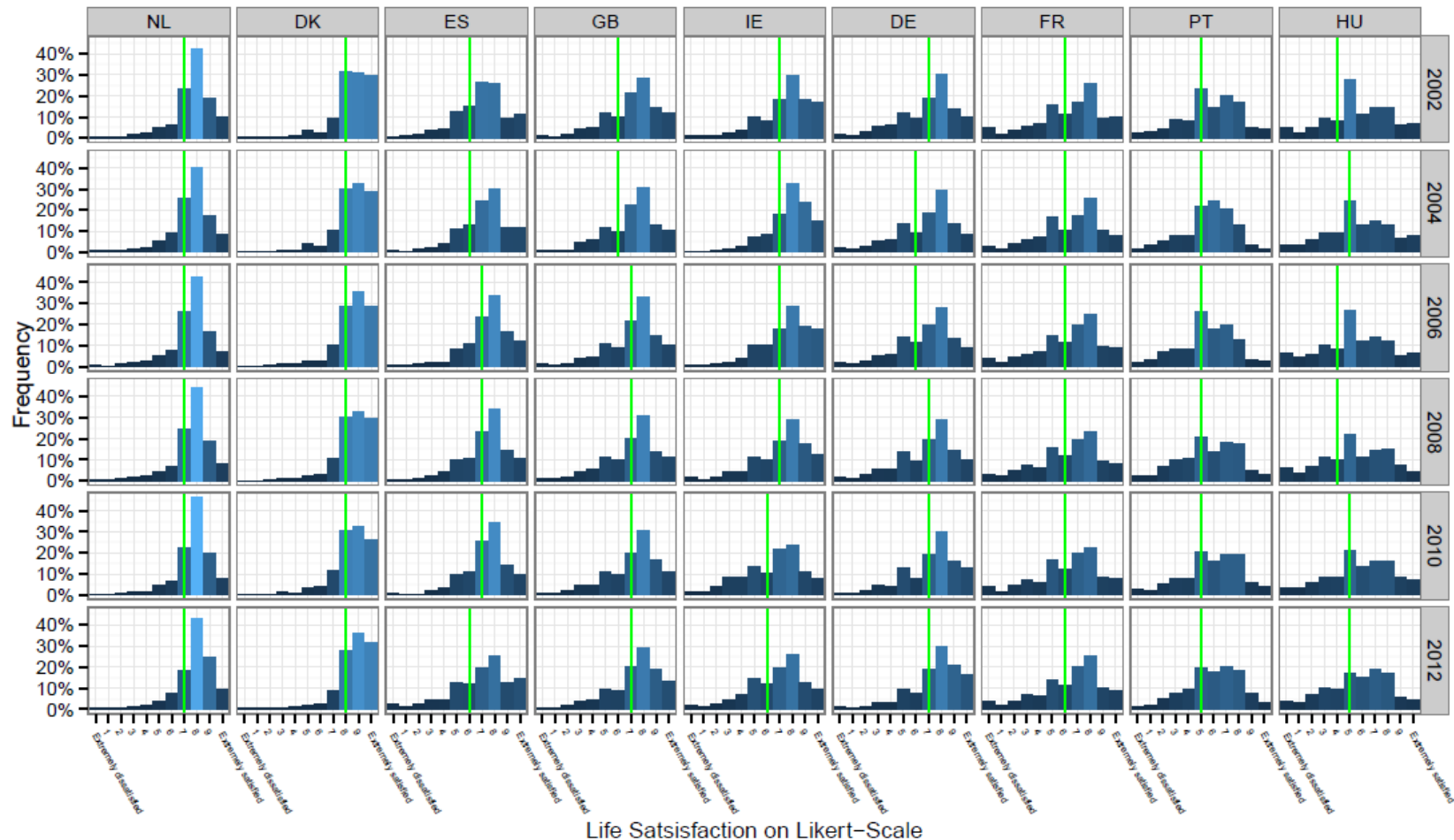
- Dominance Comparisons in **other studies**

Authors	Context	First Order Dominance	Allison-Foster Dominance
Allison-Foster (2004)	Self-assessed health (SAH) in 25 US states	199/600 (33%)	55/600 (9%)
Abul Naga/Yalcin (2008)	SAH in Swiss Cantons		12/42 (28%)
Dutta/Foster (2013)	Happiness in USA	AFD more frequently than FOD!	
Madden (2011)	wellbeing in Ireland for different life domains	Considerable variations in proportions of FOD/AFD	

Explanations: → Role of the Scale and Median Category
→ Dilemma of dense Likert-Scales!

Country-Year-Distributions

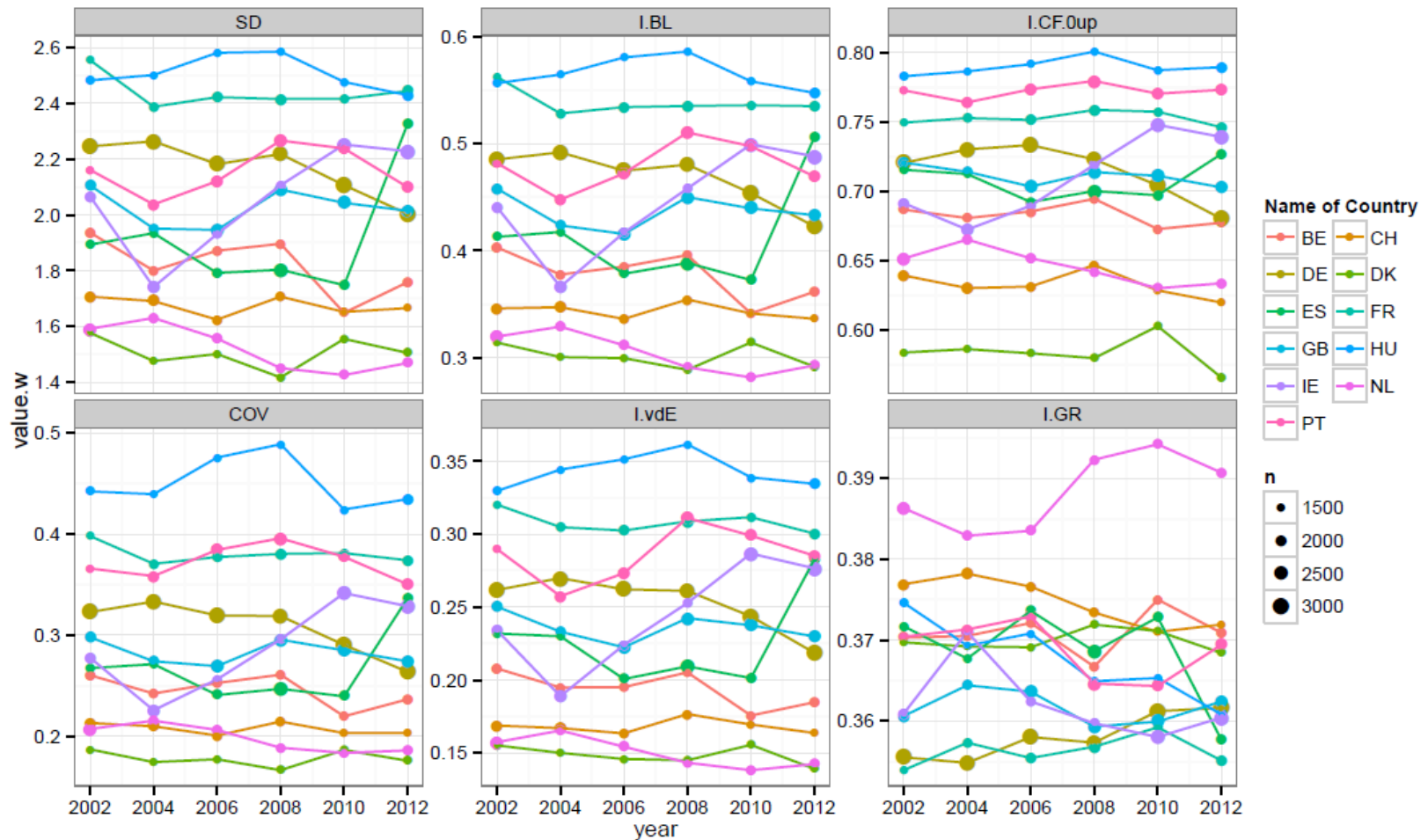
Figure 1:
Histograms of Life Satisfaction by Country and Year.



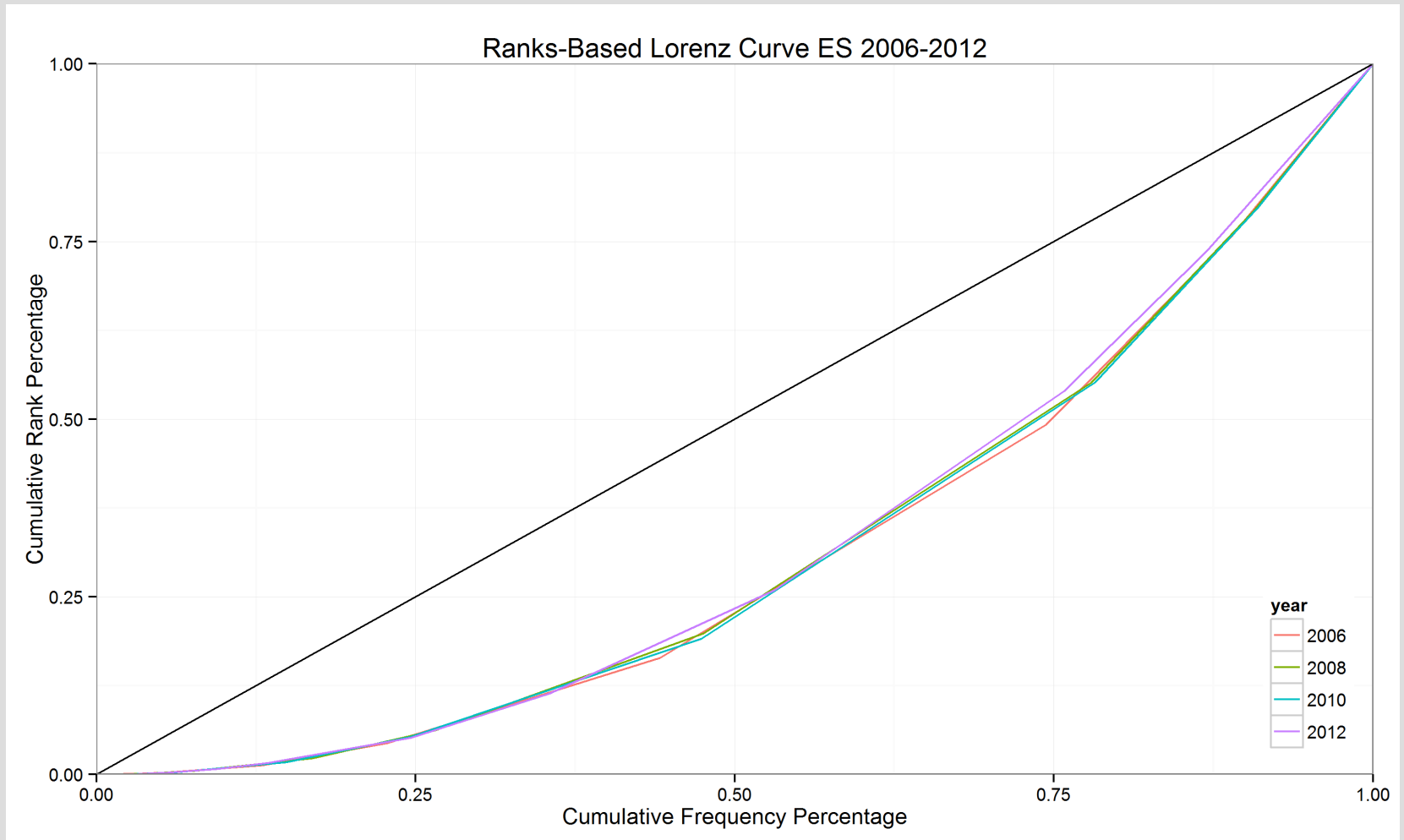
Note: Green Line indicates median category.

Comparing Measures and Countries

Figure 2:
Evolution of Inequality in European Countries.

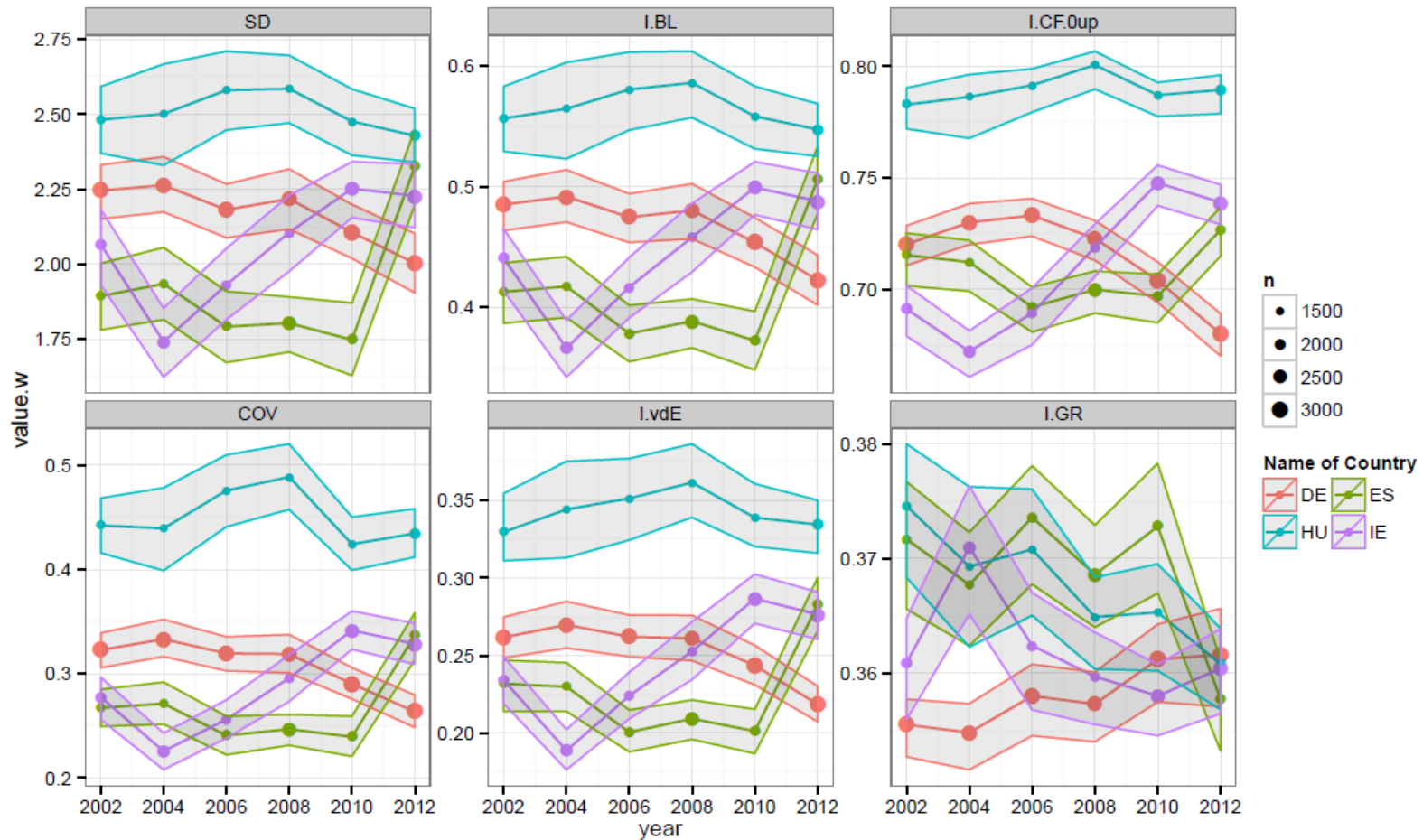


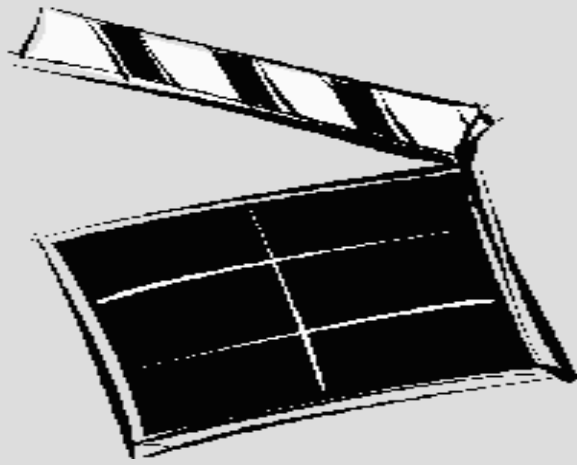
The Ranks-Based Gini – Sensitivity Issue?



Effects of Crisis?

Figure 3:
Bootstrapped Confidence Intervals for Inequality Measures.





Conclusions

Important findings

- Philosophy of Science result: Methods are around but not applied in all scientific communities
- Methods can still be improved (limited applicability of AF-ordering) – nonparametric approaches?
- AF-ordering can be criticized for methodological (median not necessarily unique) and ontological (idea of transfer) reasons
- Measures by Blair/Lacy (2000) and Van der Eijk (2001) satisfy median-preserving spread by Allison/Foster (2004)
→ inequality measures!
- To date: Most sensible approach is combining dominance analysis and Ordinal measures from different families
- Dominance Analysis restrictive if Likert-Scale is dense

Important findings

- Similar findings for cardinal and ordinal measures
- Results for ranks-based measures differ significantly
- Empirically:
 - Inequality in HU hardly changed (2002-2012)
 - Increase in Inequality for IE and ES
 - Decrease in Inequality for DE (absolute vs. relative booms!?)

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